15.2 Exercises

1–2  Find \( \int_0^2 f(x,y) \, dx \) and \( \int_0^2 f(x,y) \, dy \).

1. \( f(x,y) = 12x^2y^3 \)
2. \( f(x,y) = y + xe^y \)

3–14  Calculate the iterated integral.

3. \( \int_0^2 \int_0^1 (6x^3y - 2x) \, dy \, dx \)
4. \( \int_0^\pi \int_0^1 (4x^2 - 9x^2y^2) \, dy \, dx \)
5. \( \int_0^2 \int_0^1 y^3e^{4x} \, dy \, dx \)
6. \( \int_0^{\pi/2} \int_0^1 \cos y \, dx \, dy \)
7. \( \int_0^3 \int_0^1 (y + y^2 \cos x) \, dx \, dy \)
8. \( \int_0^3 \int_1^1 \ln y \, dx \, dy \)
9. \( \int_0^1 \int_0^1 \left( \frac{x}{y} + \frac{y}{x} \right) \, dy \, dx \)
10. \( \int_0^1 \int_0^1 e^{-y^2} \, dx \, dy \)
11. \( \int_0^1 \int_0^1 x^2 \, dx \, dy \)
12. \( \int_0^1 \int_0^1 xy \sqrt{x^2 + y^2} \, dx \, dy \)
13. \( \int_0^2 \int_0^2 r \sin^2 \theta \, dr \, d\theta \)
14. \( \int_0^2 \int_0^2 \sqrt{5 + r} \, ds \, dt \)

15–22  Calculate the double integral.

15. \( \int_A \sin(x - y) \, da \), \( R = \{(x,y) \mid 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2\} \)
16. \( \int_A (y + xy^2) \, da \), \( R = \{(x,y) \mid 0 \leq x \leq 2, 1 \leq y \leq 2\} \)
17. \( \int_A \frac{xy^2}{x^2 + 1} \, da \), \( R = \{(x,y) \mid 0 \leq x \leq 1, -3 \leq y \leq 3\} \)

23–24  Sketch the solid whose volume is given by the iterated integral.

23. \( \int_0^1 \int_0^1 (4 - x - 2y) \, dy \, dx \)
24. \( \int_0^1 \int_0^1 (2 - x^2 - y^2) \, dy \, dx \)

25. Find the volume of the solid that lies under the plane \( 4x + 6y - 2z + 15 = 0 \) and above the rectangle \( R = \{(x,y) \mid -1 \leq x \leq 2, -1 \leq y \leq 1\} \).

26. Find the volume of the solid that lies under the hyperbolic paraboloid \( z = 3y^2 - x^2 + 2 \) and above the rectangle \( R = [-1, 1] \times [1, 2] \).
27. Find the volume of the solid lying under the elliptic paraboloid \( z = \frac{x^2}{4} + \frac{y^2}{9} \) and above the rectangle \( R = [-1, 1] \times [-2, 2] \).

28. Find the volume of the solid enclosed by the surface \( z = 1 + e^x \sin y \) and the planes \( x = 1, y = 0, y = \pi \), and \( z = 0 \).

29. Find the volume of the solid enclosed by the surface \( z = x \sec^2 y \) and the planes \( z = 0, x = 0, x = 2, y = 0 \), and \( y = \pi/4 \).

30. Find the volume of the solid in the first octant bounded by the cylinder \( z = 16 - x^2 \) and the plane \( y = 5 \).

31. Find the volume of the solid enclosed by the paraboloid \( z = 2 + x^2 + (y - 2)^2 \) and the planes \( z = 1, x = 1, \) \( x = -1, y = 0, \) and \( y = 4 \).

32. Graph the solid that lies between the surface \( z = \frac{xy}{x^2 + 1} \) and the plane \( z = x + 2y \) and is bounded by the planes \( x = 0, x = 2, y = 0, \) and \( y = 4 \). Then find its volume.

CAS 33. Use a computer algebra system to find the exact value of the integral \( \int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} \, dx \, dy \) and \( \int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} \, dx \, dy \). Do the answers contradict Fubini’s Theorem? Explain what is happening.

34. Graph the solid that lies between the surfaces \( z = e^{-z} \cos(x^2 + y^2) \) and \( z = 2 - x^2 - y^2 \) for \( |x| \leq 1 \), \( |y| \leq 1 \). Use a computer algebra system to approximate the volume of this solid correct to four decimal places.

35–36 Find the average value of \( f \) over the given rectangle.

35. \( f(x, y) = x^2 \) \( R \) has vertices \((-1, 0), (-1, 5), (1, 5), (1, 0)\)

36. \( f(x, y) = e^{\sqrt{x} + \sqrt{y}} \) \( R = [0, 4] \times [0, 1] \)

37–38 Use symmetry to evaluate the double integral.

37. \( \int_0^1 \int_1^2 \frac{xy}{1 + x^4} \, dA, \) \( R = \{(x, y) \mid -1 \leq x \leq 1, 0 \leq y \leq 1\} \)

38. \( \int_1^2 \int_1^2 \frac{x}{1 + x^4} \, dA, \) \( R = [-\pi, \pi] \times [-\pi, \pi] \)

CAS 39. Use your CAS to compute the iterated integrals

\[
\int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} \, dx \, dy \quad \text{and} \quad \int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} \, dx \, dy
\]

For single integrals, the region over which we integrate is always an interval. But for double integrals, we want to be able to integrate a function \( f \) not just over rectangles but also over regions \( D \) of more general shape, such as the one illustrated in Figure 1. We suppose that \( D \) is a bounded region, which means that \( D \) can be enclosed in a rectangular region \( R \) as in Figure 2. Then we define a new function \( F \) with domain \( R \) by

\[
F(x, y) = \begin{cases} 
  f(x, y) & \text{if (x, y) is in } D \\
  0 & \text{if (x, y) is in } R \text{ but not in } D
\end{cases}
\]