Question 1.
1. Prove that:
   a. \( P(A \mid B \land A) = 1 \)
   b. \( P(X \land Y \mid E) = P(X \mid Y \land E).P(Y \mid E) \)
   c. \( P(Y \mid X \land E) = P(X \mid Y \land E).P(Y \mid E)/P(X \mid E) \)
      (the conditionalized version of Bayes' rule)

2. Show that the statement
   \( P(A \land B \mid C) = P(A \mid C).P(B \mid C) \)
   is equivalent to either of the statements
   \( P(A \mid B \land C) = P(A \mid C) \) and \( P(B \mid A \land C) = P(B \mid C) \)

Question 2.
Suppose you are given a coin that lands heads with probability \( x \) and tails with probability \( 1 - x \). Are the outcomes of successive flips of the coin independent of each other given that you know the value of \( x \)? Are the outcomes of successive flips of the coin independent of each other if you do not know the value of \( x \)? Justify your answer.

Question 3.
Orville, the robot juggler, drops balls quite often when its battery is low. In previous trials, it has been determined that the probability that it will drop a ball when its battery is low is 0.9. On the other hand when its battery is not low, the probability that it drops a ball is only 0.02. The battery was recharged not so long ago, so there is only a 8% chance that the battery is low. A robot observer with a slightly unreliable robot observation system sends the information that Orville dropped a ball. The reliability of the robot observer is described by the following probabilities:
   \( P(\text{observer reports Orville dropped ball} \mid \text{Orville dropped ball}) = 0.8 \)
   \( P(\text{observer reports Orville dropped ball} \mid \text{Orville did not drop ball}) = 0.1 \)

   a. Draw the Bayesian network.
   b. Calculate the probability that the battery is low given the information of the robot observer.

Question 4.
Propose a voting model of 10 voters for the concept “hot weather temperature” in Vietnam and derive the corresponding fuzzy set. Assume a discrete temperature range from 0°C to 40°C for the domain of the fuzzy set.